On the upper bound of the maximal absolute projection constant

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Abstract

Let X be a Banach space over $\mathbb{K}$, where $\mathbb{K} = \mathbb{R}$ or $\mathbb{K} = \mathbb{C}$. Let $Y \subset X$ be a subspace. By $\mathcal{P}(X, Y)$ denote the set of all linear and continuous projections from X onto Y, recalling that an operator $P\colon X \to Y$ is called a *projection* onto Y if $P|_Y = \mathrm{Id}_Y$. We define the *relative projection constant* of subspace Y of space X by

$$\lambda(Y, X) := \inf\{\|P\| :\ P \in \mathcal{P}(X, Y)\}.$$

Now we can define the *absolute projection constant* of Y by

$$\lambda(Y) := \sup\{\lambda(Y, X) : Y \subset X\}. \tag{0.1}$$

The ultimate goal of researchers in this area is to determine the exact value of *maximal absolute projection constant*, which is defined by

$$\lambda_{\mathbb{K}}(m) := \sup\{\lambda(Y) :\ \dim(Y) = m\}.$$

In 1994, H. König and N. Tomaczak-Jaegermann stated the following estimation.

Theorem 1 (stated in [3]; proved in [1]). *Let $m > 1$ then*

i) $\lambda_{\mathbb{R}}(m) \leq \frac{2}{m+1}\left(1 + \frac{m-1}{2}\sqrt{m+2}\right)$

ii) $\lambda_{\mathbb{C}}(m) \leq \frac{1}{m}\left(1 + (m-1)\sqrt{m+1}\right)$.

Unfortunately, their proof is based on an erroneous lemma, as was pointed out in [2]. In this talk, we will present the correct proof of the latter. Moreover relying on this result we provide the exact values of $\lambda_{\mathbb{K}}(m)$ in cases where the maximal equiangular tight frame exists in $\mathbb{K}^m$.

References

[1] B. Deregowska, B. Lewandowska, *On the upper bound of the maximal absolute projection constant providing the simple proof of Grunbaum conjecture* arXiv. https://doi.org/10.48550/arXiv.2206.09454, (2022).

[2] B. L. Chalmers, G. Lewicki, *Three-dimensional subspace of $l_\infty^{(5)}$ with maximal projection constant,* J. Funct. Anal. 257/2 (2009), 553–592.

[3] H. König, N. Tomczak-Jaegermann, *Norms of minimal projections,* J. Funct. Anal. 119/2 (1994), 253–280.